JOURNAL OF GUIDANCE, CONTROL, AND DYNAMICS
Vol. 16, No. 4, July–August 1993

Passive Damping for Robust Feedback Control of Flexible Structures

Richard Gueler*
Matsushita Electric Industrial Company, Ltd., Osaka 571, Japan
and
Andreas H. von Flotow† and David W. Vos‡
Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

This paper investigates the benefits of passive damping in single-input single-output (SISO) and multi-input multi-output (MIMO) feedback controlled structures. Theoretical formulations are derived verifying improved stability robustness characteristics for simple controlled structures on phase margin, gain margin, and root locus properties of a SISO system. Control design techniques for closed-loop bandwidths beyond the first modal frequency require accurate knowledge of the structural dynamics, particularly at crossover. The use of passive damping in the structural design allows for a greater margin of error in pole-zero cancellation at crossover, thus improving the stability robustness. Minimum levels of required passive damping are derived for robust control of uncertain structures. The derivations are extended to suggest application to MIMO systems. Robustness improvements are quantified in case studies for an 8th-order SISO example and an 18th-order MIMO example and compared to the simple derivations.

Introduction

ONE important structural parameter that has been largely neglected in the research literature on control of flexible structures is passive damping. Only recently have passive damping techniques received attention in improving the characteristics of controlled structures.[1,2,4,5,8,9,13] One key advantage of increasing the amount of passive damping in a feedback controlled structure is the improved stability robustness characteristics.[6,10,13] Reference 13 points out that for high bandwidth control of flexible structures, pole-zero cancellation is necessary. Uncertainty in the modeled dynamics can cause a pole-zero flip in the open-loop transfer function, leading to large local phase uncertainty and resulting in an unstable closed-loop system. The required level of damping to prevent this result depends on the pole-zero uncertainty and on the phase margin of the compensated system. Although much research is currently taking place in the area of robust control design, very little emphasis is placed on how to design structures so that they are inherently robust under closed-loop control. Passive damping appears to be the most important robustness parameter of the structure.

This paper addresses the problem statement, "The control bandwidth must include many poorly modeled, lightly damped, closely spaced modes." This problem statement is figuratively depicted in Fig. 1a. The fundamental point made in the paper is that no linear time invariant compensation exists that robustly (stability robustness as opposed to performance robustness) achieves what is suggested by Fig. 1a unless a significant amount of passive damping is present. The required level of passive damping is sketched in Fig. 1b. Arguments are presented that quantify required amounts of passive damping for stability robustness on simple single-input single-output (SISO) examples. Design case studies are then used to test these predictions for SISO and multi-input multi-output (MIMO) controlled structures.

Received Sept. 6, 1991; revision received Sept. 20, 1992; accepted for publication Oct. 5, 1992.

*Engineer, Production Engineering Laboratory.

†Associate Professor, Department of Aeronautics and Astronautics. Member AIAA.

‡Graduate Research Assistant, Department of Aeronautics and Astronautics.

Robustness Benefits of Passive Damping

Gain Stabilization Beyond the Bandwidth

The basic idea is that poorly modeled or unmodeled flexible modes beyond the bandwidth must be gain stabilized. Because the loop gain of a flexible structure is maximized near each resonance at a value inversely proportional to damping ratio, the conclusion emerges that an undamped flexible mode can never be gain stabilized. How much damping ratio is required to ensure gain stabilization of modes beyond the control bandwidth depends on the gain rolloff of the loop, the spectral separation between the modal natural frequency and the control bandwidth, and the modal participation or residue. The relation is rather obvious and has been reported elsewhere.[3,7,12,13] This requirement leads to the gain stabilization curve sketched in Fig. 1b.

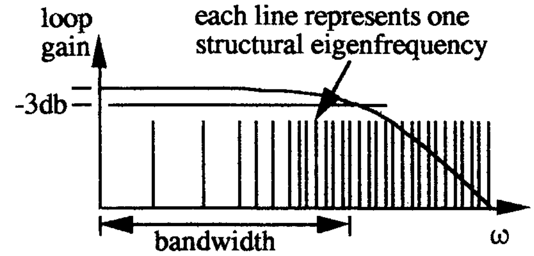

Fig. 1a Figurative depiction of problem statement for bandwidth to include many poorly modeled, lightly damped, closely spaced modes.

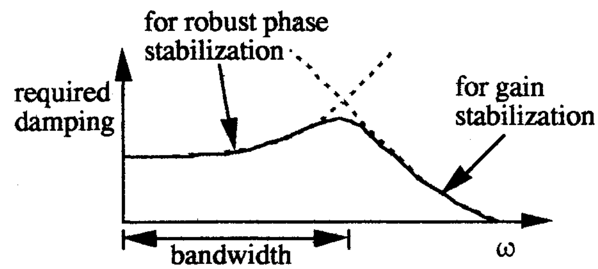

Fig. 1b Required level of passive damping to meet problem specification.

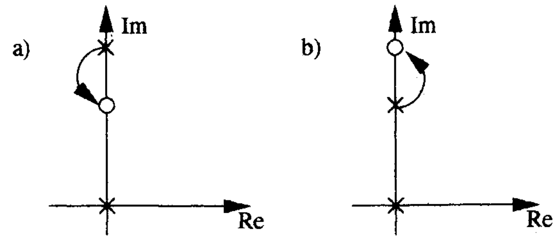

Fig. 2 Departure angles of root locus of a single oscillatory system as a result of uncertainty in pole location.

Phase Stabilization Within the Bandwidth

Robust Pole-Zero Cancellation

To achieve high bandwidth control of a flexible structure, the unwanted dynamics within the control bandwidth of the plant must be compensated, particularly at crossover. Except in special cases (such as damping with collocated sensor-actuator pairs), this must be accomplished by notch filtering the control signals at the structure's natural frequencies, resulting in cancellation of plant poles by compensator zeros and cancellation of plant zeros with compensator poles. Thus an accurate model of the structure is needed to determine its poles and zeros for the design of the compensator. For undamped systems, uncertainty in these values can result in instability in the closed-loop system.

Figure 2 shows a simple root locus of the system with poles below or above a neighboring zero as a result of uncertainty in modeled plant dynamics. If a phase lag of -90 deg is assumed due to all other dynamics of the loop, the departure angle of the root locus is 180 deg for the pole-zero pattern of Fig. 2a, but 0 deg for the pole-zero pattern of Fig. 2b. The poles migrate to the zero approximately in semicircles as feedback gain is increased. By placing a little damping in the structure, the plant poles are shifted to the left half plane leading to a system in which exact pole-zero cancellation need not be achieved. The degree to which the closed-loop poles migrate to the right-hand side depends on the pole-zero separation. Because the poles migrate to the zeros approximately in semicircles, the amount of passive damping to assure stability robustness is given approximately by

$$\zeta = \frac{|\omega_z - \omega_n|}{\omega_z + \omega_n} \tag{1}$$

where ω_z is the zero natural frequency and ω_n is the pole natural frequency. This amount of passive damping assures that the root locus will not cross the imaginary axis for any gain value.

Phase Properties of Passively Damped SISO Plants

Assume the plant under consideration is modally sparse, and in the neighborhood of each oscillatory mode is well represented by the following transfer function

$$G(s) = \frac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{2}$$

where ω_n is its natural frequency and ζ is the passive damping ratio. For this system, the phase angle $\theta(\omega)$ at any frequency is given by

$$\theta(\omega) = -\tan^{-1}\frac{2\zeta\omega_n\omega}{\omega_n^2 - \omega^2} \tag{3}$$

It can be shown that at resonance ($\omega_n = \omega$), the change in phase angle with respect to frequency is given by

$$\frac{d\theta}{d\omega} = \frac{-1}{\zeta\omega_n} \tag{4}$$

which says that the phase change at resonance is sharp for low damping (see Fig. 3). If the uncertainty in the eigenfrequency is given by $\delta\omega = \omega_n - \omega_{\text{actual}}$, then a first-order approximation of the uncertainty in phase angle near resonance is given by

$$\delta\theta = -\delta\omega/\zeta\omega_n \tag{5}$$

Thus the uncertainty in phase of the plant, given an uncertainty in natural frequency, is inversely proportional to the damping.

By observing the phase excursion in imperfect pole-zero cancellation, the degradation of closed-loop stability can be determined. Figure 4 shows the phase excursion as a result of the actual plant natural frequency being less than the modeled frequency ($\omega_{\text{actual}} < \omega_n$), which introduces a local phase lag. Given an undamped structure with imperfect pole-zero cancellation, the local phase excursion is 180 deg. The introduction of damping reduces this phase excursion.

For a local phase margin of $\delta\theta_{\text{pm}}$, which is the amount of additional phase lag needed to induce instability at that frequency, the permissible amount of uncertainty in the plant natural frequency is given by

$$\delta\omega \le \delta\theta_{\text{pm}}\zeta\omega_n \tag{6}$$

Thus it is sufficient to say that the amount of damping needed, given a pole-zero mismatch of $\delta\omega$ and a nominal local phase margin of $\delta\theta_{\text{pm}}$, is

$$\zeta \ge \frac{1}{\delta\theta_{\text{pm}}}\frac{\delta\omega}{\omega_n} \tag{7}$$

This expression leads to the phase stabilization curve sketched in Fig. 1b.

Simple Structure with Proportional Derivative Control

Hughes and Abdel-Rahman[7] and Spanos[12] have reported related studies on classical feedback compensation of flexible structures with simple models of flexibility. The following example examines the benefits of passive damping on a simple structure with proportional derivative (PD) control. These results are taken from Spanos.[12] Stability bounds are determined based on structural configurations and performance limits.

Consider a structure with one rigid body mode and one flexible mode at frequency ω_n. The transfer function representing the structure is given by

$$\frac{y(s)}{u(s)} = \frac{1}{Js^2} + \frac{\phi_{in}\phi_{jn}}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{\alpha_n(s^2 + 2\zeta\omega_n s + \omega_n^2)}{Js^2(s^2 + 2\zeta\omega_n s + \omega_n^2)} \tag{8}$$

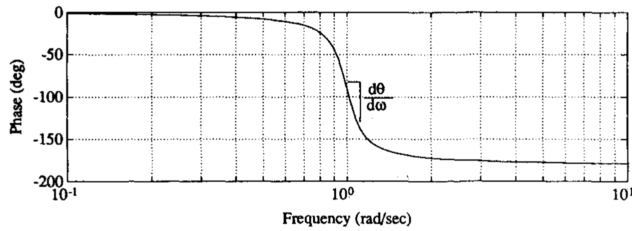

Fig. 3 Bode plot illustrating phase change at resonance.

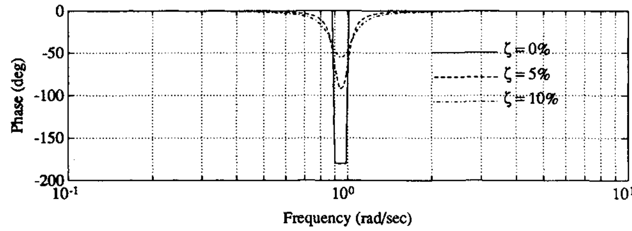

Fig. 4 Bode plot illustrating phase excursion due to imperfect (by 10%) pole-zero cancellation for different amounts of passive damping.

where $y(s)$ is the measured position of the structure and $u(s)$ is the control effort applied to the structure. The inertia of the structure is given by J with a modal damping level of ζ and a natural frequency of ω_n. The term α_n represents the modal participation coefficient of the mode defined as

$$\alpha_n = 1 + J\phi_{in}\phi_{jn} \tag{9}$$

where ϕ is the eigenvector (mode shape) of a given mode normalized to unit mass. The modal participation coefficient reflects the mass participation in a given mode.

If the mode is minimum phase ($\alpha \geq 0$) and lightly damped, it can be shown that

$$\alpha_n = \omega_n^2/\omega_z^2 \tag{10}$$

where ω_z is the zero frequency, resulting in

$$\frac{\omega_z - \omega_n}{\omega_n} = \frac{1}{\sqrt{\alpha_n}} - 1 \tag{11}$$

Separation between poles and zeros is thus inversely proportional to the mass participation in the mode defined by α_n. A weakly participating mode ($\alpha_n \approx 1$) leads to near cancellation of the plant pole-zero pair.

Under PD control, for a given closed-loop bandwidth ω_{cl} and ζ_{cl}, as well as a structural natural frequency ω_n and modal damping ζ, Spanos[12] reports conditions on the modal participation coefficient α_n to maintain stability. These stability boundaries are replotted in Fig. 5 given a desired closed-loop damping ratio of $\zeta = 1/\sqrt{2}$.

By using the algebraic results of reference,[12] which are represented in Fig. 5, and applying Eq. (11), a relationship can be established between the maximum allowable plant pole-zero separation and minimum modal damping for stable control of arbitrary bandwidth as shown in Fig. 6. The figure shows an inverse linear proportionality between modal damping needed to stabilize a controlled structure and pole-zero separation. In comparison with the semicircle assumption and the phase margin assumption of Eq. (6), the results may differ by as much as a factor of two.

Directional Properties of Passively Damped MIMO Plants

Small changes in plant variables can result in dramatic differences in the directional properties of a lightly damped plant. The addition of passive damping to a MIMO plant reduces the sensitivity of direction to plant variable changes. Consider an

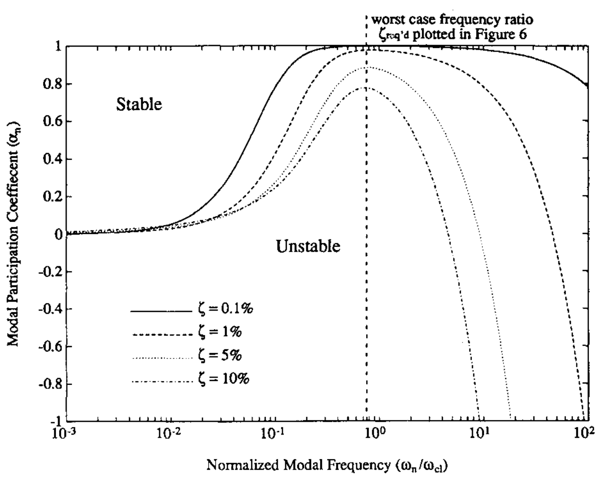

Fig. 5 Stability bounds of a single mode structure with passive damping control for various levels of modal damping given a desired closed-loop damping ratio of $\zeta = 0.7071$.[12]

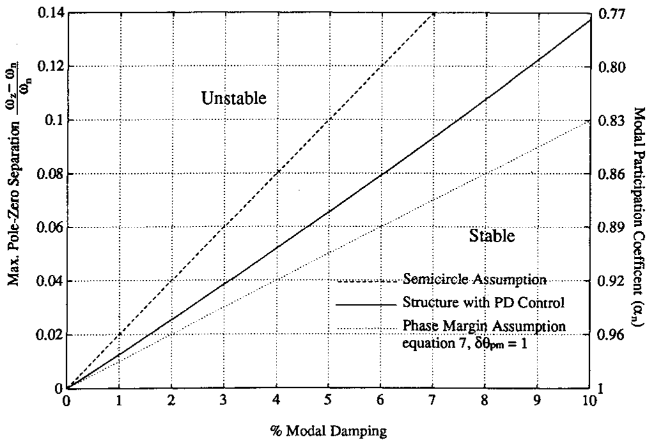

Fig. 6 Maximum allowable plant pole-zero separation to maintain stability.

n-degree-of-freedom structural model with the transfer function from input u_i to output x_j as follows:

$$H_{ji}(s) = \frac{x_j(s)}{u_i(s)} = \sum_{r=1}^{n} \frac{\phi_{jr}\phi_{ri}}{s^2 + 2\zeta\omega_r s + \omega_r^2} \tag{12}$$

Notice that the poles of the plant are given by the denominator of the transfer functions, which are the same for all input to output relationships. Directional changes are based on the numerator of the transfer function, which also defines the plant zero locations. Thus uncertainty in the plant scalar zeros indicates uncertainty in the plant directions.

For a two-input two-output system, one such indicator of plant directions is the ratio of the ouptut states given a specified input. Consider a two-degree-of-freedom system. The ratio of the output states given only the input $u_1(s)$ is

$$\frac{x_1(s)}{x_2(s)} = \frac{a_1(s^2 + 2\zeta\omega_{z1}s + \omega_{z1}^2)}{a_2(s^2 + 2\zeta\omega_{z2}s + \omega_{z2}^2)} \tag{13}$$

$$a_1 = \phi_{11}\phi_{11} + \phi_{12}\phi_{21} \qquad a_2 = \phi_{21}\phi_{11} + \phi_{22}\phi_{21} \tag{14}$$

where ω_{z1} and ω_{z2} are zero frequencies of $H_{11}(s)$ and $H_{21}(s)$, respectively. The phase difference between the two outputs at a given frequency ω is then

$$\theta(\omega) = \tan^{-1}\frac{2\zeta\omega_{z1}\omega}{\omega_{z1}^2 - \omega^2} - \tan^{-1}\frac{2\zeta\omega_{z2}\omega}{\omega_{z2}^2 - \omega^2} \tag{15}$$

The frequency where rapid phase change occurs is at the location of the plant scalar zeros ($\omega = \omega_{z1}$ and $\omega = \omega_{z2}$). For small damping values, the phase gradient ($d\theta/d\omega$) at the zero frequencies is approximately

$$\left.\frac{d\theta}{d\omega}\right|_{\omega_{z1}} \approx \frac{1}{\zeta\omega_{z1}} \qquad \left.\frac{d\theta}{d\omega}\right|_{\omega_{z2}} \approx \frac{1}{\zeta\omega_{z2}} \tag{16}$$

If the uncertainty in the zero location is given by $\delta\omega_{z1} = \omega_{z1} - \omega_{z1,\text{actual}}$, then a first-order approximation of the uncertainty in phase difference between the two outputs is given by the following:

$$\delta\theta = \delta\omega_{z1}/\zeta\omega_{z1} \tag{17}$$

This equation is similar to that derived for the phase uncertainy of a SISO system [see Eq. (5)]. The uncertainty in the phase difference between the outputs of the plant (the plant directions) at the plant zero locations is inversely proportional to damping. Similar equations can be derived for output ratios as a result of inputs into the second channel and for ratios of

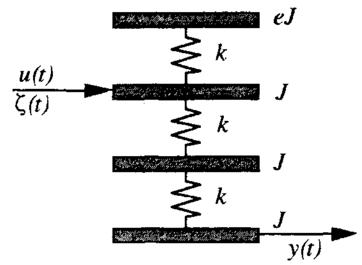

Fig. 7 Four-disk system (based on Ref. 11).

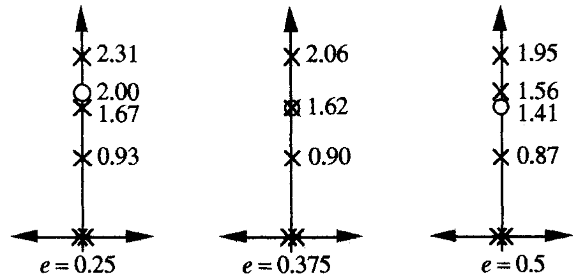

Fig. 8 Pole-zero plot of nominal and extreme uncertainty cases.

inputs while measuring only one output. This idea can be extended for higher-order MIMO systems, but the implication of uncertainty in these directions on control system robustness is not easy to quantify.

SISO Four-Disk Example

This example problem investigates the virtues of passive damping on a structure consisting of four disks connected by flexible springs as shown in Fig. 7. It uses a system originally proposed by Rosenthal.[11] The objective is to control the angular position of one of the disks by applying a force on another disk given some uncertainty in the inertia in the top disks. The structure is modeled as a fourth-order system consisting of four lumped masses and three springs.

The actuator is placed such that it is at the nominal node of the second flexible mode. This results in a pole-zero cancellation in the nominal plant model making the second flexible mode uncontrollable. The uncertainty in the plant model results in a pole-zero flip as shown in Fig. 8, which makes it difficult to control near that frequency. The uncertainty in the disk inertia is given by $0.25 \leq e \leq 0.5$. A nominal value of $e = 0.375$ is used to compute the controller.

Improvements in stability robustness and performance robustness are investigated for various amounts of modal damping added to this system with H_2 control design. The system framework used to derive the controller is shown in Fig. 9. Two control designs are computed such that good command following of the fourth disk is achieved by applying an appropriate torque on the second disk. The first control design uses a low bandwidth controller ($\omega_{cl} = 0.08$ rad/s) such that the bandwidth is one decade below the first structural natural frequency. The flexible modes are gain stabilized requiring small amounts of passive damping. The second control design uses a high bandwidth controller ($\omega_{cl} = 1.2$ rad/s) that encompasses the first structural mode and lies close to the sec-ond mode. Thus an accurate plant model is needed. This example shows the need for passive damping in a phase stabilized system.

The allowable uncertainty in the inertia of the first disk (e) to maintain stability for various levels of modal damping is shown in Fig. 10. Because the low bandwidth control system used gain stabilization of the flexible modes, it was more robust than the system with the high bandwidth controller. The high bandwidth case needed significant damping to assure stability robustness. The results of the four-disk case study showed excellent correlation from the results predicted in the preceding section.

Comparison of Figs. 10 and 11 shows similarity in the effects of increased passive damping on stability and performance robustness. In this example, performance robustness was (arbitrarily) based on the allowable uncertainty in e such that the H_2 norm of the system did not vary by more than 5%. To maintain 5% performance robustness, more damping was needed than for stability robustness. Major gains in performance robustness were achieved with the addition of passive

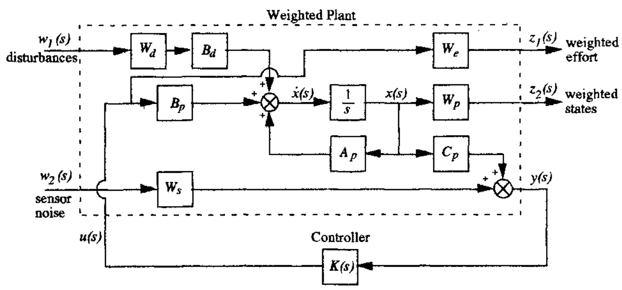

Fig. 9 Block diagram of closed-loop system.

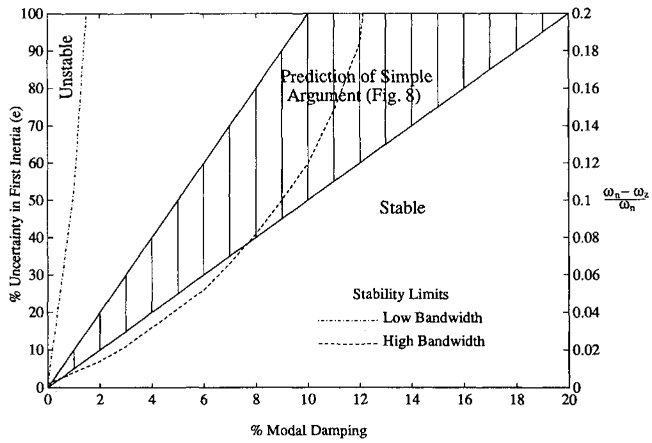

Fig. 10 Plant pole-zero mismatch and allowable uncertainty in inertia of first disk to maintain stability of system for various amounts of passive damping.

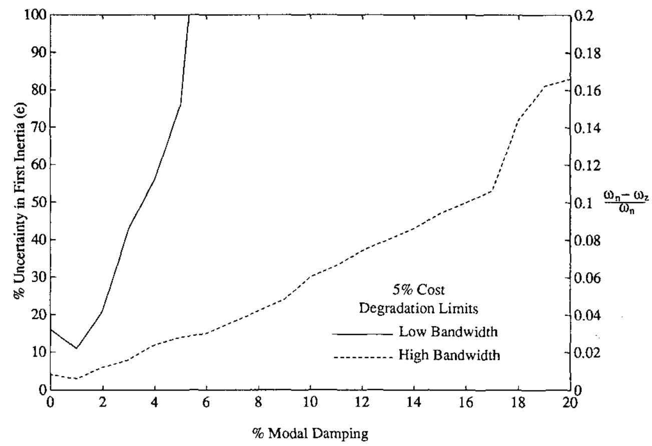

Fig. 11 Plant pole-zero mismatch and allowable uncertainty in inertia of first disk to maintain good performance of system for various amounts of passive damping.

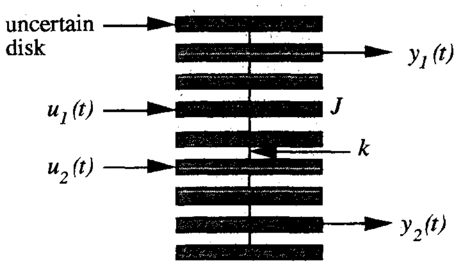

Fig. 12 Nine-disk MIMO structure.

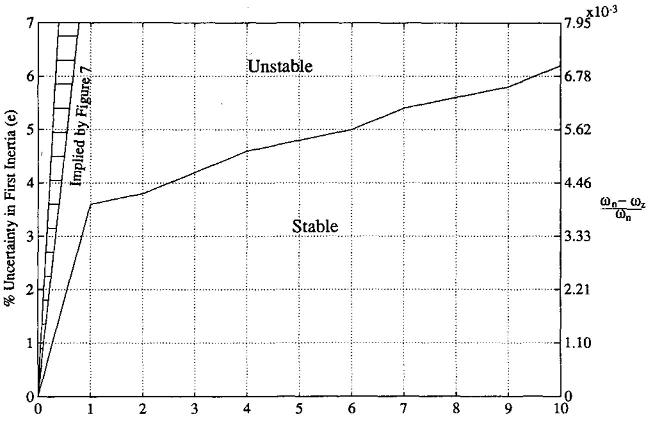

Fig. 13 Allowable uncertainty in inertia of first disk to maintain stability of nominal system for various amounts of passive damping.

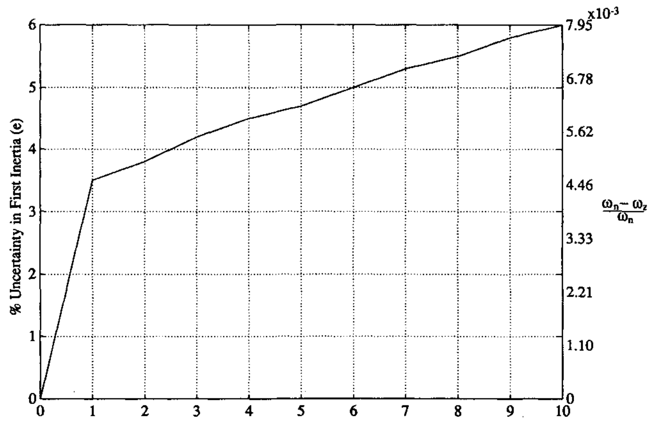

Fig. 14 Allowable uncertainty in inertia of first disk to maintain good performance of nominal system for various amounts of passive damping.

damping. The low bandwidth design again was less sensitive to uncertainties than high bandwidth design.

MIMO Nine-Disk Example

MIMO control methods such as H_2 control take advantage of the directionality of the system dynamics. Thus robustness with respect to uncertain plant directions becomes important in the control of MIMO flexible structures. In this example, passive damping effects are examined on a MIMO structure made up of a nine-disk system connected by flexible springs with two actuators and two sensors as shown in Fig. 12. The actuators are located on the fourth and sixth disks, whereas the sensors are on the second and eighth disks. The structure has nine modes. The sensors are located at the nominal nodes of the fourth mode ($\omega_n = 1$ rad/s) making it unobservable, hence active control at the frequency of the fourth mode is impossible. That is, a transmission zero lies at the frequency of the fourth mode resulting in a plant pole-zero cancellation. Any uncertainty in the model can result in a pole-zero flip which may cause instability.

As with the preceding example, this investigation examines the virtues of passive damping on the nine-disk system with MIMO control for active vibration suppression with H_2 control design. The same framework as the preceding example was used to derive the H_2 controller with the following weights:

$$W_d = I$$

$$W_s = I \times 10^{-6} I$$

$$W_e = 1 \times 10^{-6} I$$

$$W_p = \begin{bmatrix} I & \\ & 0.01 \times I \end{bmatrix} \begin{bmatrix} \Phi & 0 \\ 0 & \Phi \end{bmatrix}$$

As with the preceding example, the stability and performance robustness properties of the system are investigated given an uncertainty in the inertia of the first disk. Because of the higher complexity and pole-zero cancellation in the system, the control system is very sensitive to changes in the undamped structure. Again, good performance is based on the H_2 norm not deviating by $\pm 5\%$ from the nominal value given an uncertainty in the inertia of the first disk.

Results of this investigation using MIMO control on the nine-disk system were similar to that of the four-disk system. Again, improvements in stability and performance robustness were achieved through passive damping. The undamped MIMO nine-disk system was very sensitive to model uncertainty as shown in Figs. 13 and 14. The undamped case could not tolerate any uncertainty. But for both stability and performance robustness, as much as a 6% uncertainty was tolerable in the inertia of the first disk given 10% modal damping. With just 1% modal damping, a 3.5% uncertainty in the inertia was tolerable.

Conclusion

This paper has argued simply, but quantitatively, that a critical level of passive damping can be specified that will permit robust noncollocated linear time invariant control of structural dynamics with the control bandwidth encompassing many flexible eigenfrequencies. This level of required passive damping is proportional to model uncertainty and is greatest at crossover. Figure 1 perhaps summarizes the thesis of this paper most succinctly.

The results from the design studies show much similarity to those predicted theoretically. The correspondence with the simple argument was best for the SISO case study. The MIMO case study suggested that the MIMO problem is even more sensitive than the SISO problem, and that even more passive damping is required to ensure robust MIMO control of uncertain structural dynamics with noncollocated sensor-actuator sets. The results reported in the case studies were derived using standard H_2 control synthesis techniques, ignoring plant uncertainty.

It is instructive to compare typical levels of passive damping in aerospace structures (approximately 1% of critical) to typical levels of uncertainty in modal eigenfrequency and zero locations (several percent). The conclusion that emerges is that robustness control of structural dynamics will not be possible without strong augmentation of passive damping levels.

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